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R & D NOTES

The Axial Velocity of a Submerged Axially Symmetrical Fluid Jet

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To mix liquids in very large tanks it is often convenient to use a submerged jet of liquid formed by discharge through a pipe or nozzle. Problems frequently arise in the filling of tanks to mix the inflow with the tank contents. Tanks holding up to 159 000 m³ (one million barrels) can be blended with inflow if the inflow is properly positioned and of sufficient velocity so that a jet is formed which will entrain surrounding fluid and mix in the turbulent core of the jet.

The axial velocity of such a submerged jet will determine the power required and the rate of blending or entrainment; thus, a knowledge of the velocity of submerged jets as they progress is necessary for the design of such systems. The data herein presented have been used successfully in the design of the filling-mixing systems for tanks ranging in size from 37.85 to 159 000 m³ (Sarsten, 1972). See Rushton (1954), Fossett and Prosser (1949), Folsom and Ferguson (1949).

A fluid jet whose cross section is circular will entrain surrounding fluid as it flows from and along the axis of the nozzle. The axial velocities of the jet will increase as the jet flows from the nozzle and as its cross section increases owing to entrainment of surrounding fluid. The momentum of the jet will remain constant as it expands by entrainment.

The distance from the nozzle to a position along the axis of flow is designated as x , and the diameter of the nozzle is D_0 . The velocity at the jet center line, or axis, is u_x at point x , and u_0 is the average velocity at the nozzle.

A large number of investigations have been made and reported in the literature on the behavior of air and gas jets in air under both isothermal and nonisothermal conditions. Those which report original data for isothermal conditions and for jet lengths between 20 and 100 jet diameters (x/D_0), are listed in the references. Some papers have been published on the flow behavior of liquid jets in liquids (Donald and Singer, 1959; others in the references). Of a number of books on the mechanics of

fluid streams, the theory of the flow characteristics of submerged jets is perhaps best developed and covered by G. N. Abramovich in *The Theory of Turbulent Jets* (1963). Liquids in large tanks (for example, tank volumes up to 159 000 m³ of liquid) are mixed while filling by pumping through a pipe or nozzle opening, or by the use of side entering propeller mixers (Rushton, 1954; Fossett and Prosser, 1949).

In the following discussion the theory by Abramovich will be combined with experimental results in the literature to justify a new relation by which jet velocities for various liquids can be predicted.

Abramovich shows the well-established theoretical equation for the ratio of maximum center-line velocity u_m (here the same as the notation u_x , above) to the velocity at the nozzle u_0 :

$$u_x/u_0 = u_r = (0.48/a) (D_0/x) \quad (1)$$

where u_r is the ratio of center-line jet to average velocity at D_0 . From this, the velocity decrease at distances x can be calculated if the constant a is known. Equation (1) also appears in the literature where $(0.48/a)$ is expressed as a constant. The development of these equations is based on the well-substantiated experimental fact that the velocity profile of expanding submerged fluid jets maintains similarity of flow velocities across the jet for distances $(x/D_0) > 6$. The length over which a jet will expand by entraining surrounding fluid is a function of its velocity and Reynolds number.

The constant a has been evaluated in the experiments of Trupel (1915), Zimm (1921) and by Prandtl, Turkus, Syrkina and Lyakhovsky, as summarized by Abramovich (1963). The variation of a appears to be dependent upon the uniformity of velocity of the initial jet flow and of the turbulence. Abramovich gives values of a of 0.066, 0.070 and 0.076. He further states that values of a will increase as the velocity profile in the nozzle becomes less uniform. It is well known that lower Reynolds numbers in a pipe or nozzle result in lower velocity uniformity. Thus, for real liquid flows, the constant a should be equal to the values given by Abramovich only at the same Reynolds number.

Corrsin (1943) gives data which also agree generally with those cited by Abramovich, except that the values of a are lower. Nottage et al. (1952) and Tuve and Priester (1944) also give data for air which lead to values of a which are both higher and lower than given by Abramovich. They show that the constant will vary for different limits of initial velocity. The Cleaves and Boelter (1947) data and references they cite also lead to values of a which are both higher and lower than 0.070. Thus, it appears that the constant a is not independent of Reynolds number but is a function of the turbulence at the initial cross section of the jet as Abramovich states (1963). Since the Reynolds number is a measure of the ratio of turbulent to viscous forces, it follows that the effect of Reynolds number of Equation (1) should be determined.

Data on the velocities and spread of water and other fluid jets have been reported by Binnie (1942) and by Donald and Singer (1959). Folsom and Ferguson (1949) have theorized on the jet behavior of liquids using literature data. Fossett and Prosser (1949) give data for mixing of liquids with small diameter jets in large tanks. Rushton (1954) shows comparisons of jet mixing in hydrocarbon storage tanks with propeller type mixers and jet flow from pipes and nozzles.

When the data on jet behavior of liquids in liquids are compared with those of gases, it is evident that the entrainment and velocity distributions are functions of the Reynolds number.

Donald and Singer report their data as a function of the kinematic viscosity and suggest that the expansion of a jet and its velocity are related to the Reynolds number. When all of the original data cited in the twenty-one relevant references for air and other fluids are compiled and examined, it is clear that there is a relatively simple logarithmic relation between the ratios (u_x/u_0) , (x/D_0) , and the Reynolds number $(D_0 u_0/\nu)$ of the jet at its origin.

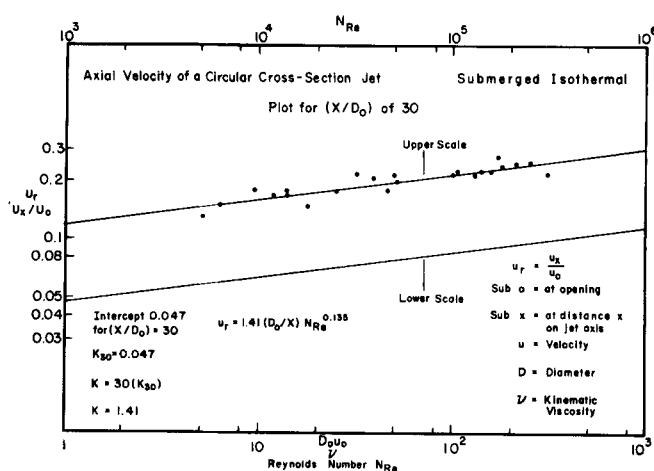


Figure 1. Axial velocity of a submerged isothermal circular cross-section jet related to the Reynolds number.

The key to relating the center-line velocities of a jet u_x to the Reynolds number is found in the work of Donald and Singer. Their data establish the angle of expansion of a submerged fluid jet for hydrogen, air, sugar-water solution and water. For these fluids they covered, a kinematic viscosity range (ν , in Stokes at 18°C) of hydrogen at 1.030 to water at 0.010, a one-hundred fold range, using Reynolds numbers up to 3.56×10^4 . They calculate a dependence of the half angle of the jet ($\theta/2$) on the kinematic viscosity of the fluid raised to the 0.133 power. A refinement of their fluid physical properties results in a better plot of their data, giving a slightly different value of 0.135 for the exponent on the viscosity. Accordingly, the Donald and Singer data result in the half angle jet expansion equation for their conditions of

$$\tan \theta/2 = 0.238 \nu^{0.135} \quad (2)$$

A recapitulation of the theory and experimental information on submerged isokinetic and axially symmetrical turbulent jet expansion may be summarized as follows:

1. Velocity profiles of cross section perpendicular to the jet axis are similar.
2. Total momentum of fluid is constant at all cross sections of isothermal jets.
3. Center-line (or axis) velocities decrease inversely with the ratio of distance to diameter of the circular pipe or nozzle from which the jet emerges, provided that the distance is greater than about $6(x/D_0)$.
4. Jets may entrain for an axial distance of 100 nozzle diameters (x/D_0) depending on the initial jet velocity, and this entrainment causes the jet to expand to a characteristic jet angle.
5. The angle between the axis and the boundary of the jet is constant for distances above about $6 D_0$.
6. The angle of expansion of the jet is a function of the kinematic viscosity of the fluid and of the velocity of the fluid.

This last item suggests that the data and conclusion of Donald and Singer should be tested for ranging axial velocity using the many data on air already in the literature.

From the above six items it follows that anything which changes the jet expansion angle will have an inverse effect on the velocity. If the jet angle (or half angle) is proportional to $\nu^{0.135}$, then the center-line velocity, u_x , and also the velocity ratio u_r , should be proportional to $(1/\nu^{0.135})$.

Further, since it has been shown by Nottage and by Tuve that u_r is some function of initial velocity u_0 , it would appear that a Reynolds number might logically be a significant correlation parameter. Since Abramovich also states that initial turbulence affects u_r , and the Reynolds number is a well-accepted parameter to quantify turbulence, it is logical to apply the Reynolds number concept to the effect of D_0 , u_0 and ν on the velocity ratio u_r along the jet axis.

Using the value of 0.135 for the exponent on ν , the test is made using $N_{Re}^{0.135}$. This, then, leads to the test of the relation:

$$u_r = K N_{Re}^{0.135} (D_0/x) \quad (3)$$

This equation is, of course, a modification of Equation (1), where

$$a = 0.48/K N_{Re}^{0.135} \quad (4)$$

a is the theoretically derived constant and is supposedly independent of fluid properties and initial flow rate. In all these cases, the data used were for jets where velocities were measured for 25 to 100 jet diameters (x/D_0).

Data from thirteen separate investigators using air are tabulated from references numbers 2 through 22 in Table 1. The table shows the relevant data for velocity ratios at x/D_0 position of 30 and viscosities for the results reported in the references. Thus, each point in the plot (Figure 1) is for one literature reference. Column two gives the number for one point characterizing each set of the reference data at an (x/D_0) value of 30 used to correlate the various results graphically and by computer. The upper part of the table (references 2 through 22) is for air data only.

From column five it can be observed that the Reynolds number N_{Re} varies from a low of 4.6×10^2 (due to a high temperature experiment) to a high of 2.5×10^5 . The value of $N_{Re}^{0.135}$ is shown in the next column.

For the test of Equation (3), the value of (x/D_0) of 30 was chosen. This value was in the range of all but two of the references and is thought to be well representative of the experimental data. Thus, column seven gives the values of the velocity ratio u_r at a distance of 30 jet diameters from the nozzle or pipe. Incidentally, there are a few data available for air jets originating from orifices; these were not used owing to questions concerning temperature and orifice coefficients.

The last column gives values of the constant K calculated from Equation (3).

At this point it may be noted that the calculated values of K are close to each other, especially considering that the data are from many experimenters, using different experimental equipment, and over a period of years from 1915 to 1952. The average value for K by this method of determination was 1.41.

In the lower part of Table 1, the available data from Binnie and by Donald and Singer are shown. These data are for hydrogen, air, sugar-water solution and water. They each represent one velocity measurement made at less than 30 diameters but are calculated for velocity ratios at 30 diameters. To show the major effect of kinematic viscosity, the velocity ratios at 30 diameters were calculated for the Donald and Singer data using the same value of the product of $D_0 u_0$. Hence, the Reynolds number tabulated for these four points shows the effect of kinematic viscosity directly. The u_r was calculated using the value of 1.41 for K .

A plot on logarithmic coordinates (Figure 1) of the values of u_r at 30 diameters on the ordinate, vs. the Reynolds number on the abscissa as tabulated in Table 1, shows a straight line fit, and a least-squares fit of the data gives a slope of 0.135, the same as for the kinematic viscosity correlation of Donald and Singer.

The constant K_{30} was found to be 0.047, where K_{30} represents $K(D_0/x)$ in Equation (3) where (D_0/x) is 30. It follows that K will equal thirty times the intercept of 0.047, giving the value 1.41. This value of the constant K determined analytically corresponds to that shown in Table 1 as found by averaging the numerical data.

The seven points from the lower part of Table 1 show clearly the relative effect of the kinematic viscosity on u_r . Point 25 is for hydrogen where ν is 1.030, 26 is for air where ν is 0.151, 27 is for a sugar solution where ν is 0.130 (close to air) and 28 is for water where ν is 0.010. Other water points 29 and 30 show the effect of a change of the product $D_0 u_0$ on u_r .

TABLE 1. SUMMARY OF PUBLISHED DATA ON AXIAL VELOCITY OF CIRCULAR CROSS-SECTION SUBMERGED JETS

	Ref. No.	Temp., °C	Diam. D , cm	Vel. at D , cm/s	Vis. ν , Stokes	N_{Re}	$N_{Re}^{0.135}$	u_r at $x/D=30$	K , calc.
	2	18	2.54	6 200	0.151	1.05×10^5	4.76	0.23	1.45
	10	20	10.0	2 500	0.153	1.6×10^5	5.03	0.23	1.37
	10	20	10.0	2 800	0.153	1.8×10^5	5.11	0.24	1.41
	5	20	4.0	5 400	0.153	1.4×10^5	4.96	0.23	1.39
	4	22	2.87	662	0.156	1.2×10^4	3.56	0.17	1.43
	4	700	2.87	1 715	10.65	4.6×10^2	2.29	0.11	1.44
	6	18	2.54	1 000	0.151	1.4×10^4	3.62	0.18	1.49
	6	18	7.62	2 050	0.151	1.0×10^5	4.75	0.22	1.39
	10	20	10.0	2 660	0.153	1.7×10^5	5.08	0.27	1.59
	11	18	12.7	382	0.151	3.2×10^4	4.05	0.22	1.63
	12	18	15.25	51	0.151	5.1×10^3	3.15	0.13	1.24
	12	18	15.25	178	0.151	1.8×10^4	3.76	0.15	1.20
	12	18	15.25	254	0.151	2.5×10^4	3.92	0.18	1.38
	12	18	15.25	508	0.151	5.1×10^7	4.32	0.20	1.39
	12	18	15.25	1 270	0.151	1.3×10^5	4.91	0.22	1.35
	12	18	15.25	3 050	0.151	3.1×10^5	5.48	0.22	1.21
	14	20	7.0	1 000	0.153	4.6×10^4	4.28	0.18	1.26
	15	20	7.2	450	0.153	2.1×10^5	5.21	0.25	1.44
	20	18	15.25	2 500	0.151	2.5×10^5	5.36	0.25	1.40
	21	20	0.25	4 130	0.153	6.7×10^3	3.28	0.15	1.43
	21	20	0.25	30 600	0.153	5.0×10^4	4.32	0.22	1.53
	22	20	5.0	295	0.153	9.6×10^3	3.46	0.18	1.56
	22	20	5.0	1 200	0.153	3.9×10^4	4.17	0.21	1.51
								Avg.	1.41
								Calculated	K Assumed
Water	3	13	0.661	299	0.010	2.0×10^4	3.80	0.18	1.41
Hydrogen	7	18	0.635	3 250	1.030	2.0×10^3	2.79	0.13	1.41
Air	7	18	0.635	3 250	0.151	1.4×10^4	3.63	0.17	1.41
Sugar in water	7	18	0.635	3 250	0.130	1.6×10^4	3.68	0.172	1.41
Water	7	18	0.635	3 250	0.010	2.0×10^5	5.18	0.24	1.41
		18	0.318	—	0.010	5.6×10^3	3.20	0.15	1.41
Water	7		to			to			
		18	1.59	—	0.010	4.2×10^4	4.20	0.20	1.41

From these available data on jet velocities, it appears that Equation (3) is a good and acceptable relation, where K has the value 1.41.

It is of interest to return to Equation (1) as derived by Abramovich (1963). Equation (3) differs from Equation (1) only in that $KN_{Re}^{0.135}$ has been replaced $(0.48/a)$ as shown in (4). Since Abramovich quotes a value of a equal to 0.076 for a turbulent velocity field with Reynolds numbers between 0.2×10^5 and 40×10^6 , it is interesting to use Equation (4) to calculate the Reynolds number which would give the value of 0.076 for a . If we use values of 0.076 for a and 1.41 for K , the solution of the equation results in a Reynolds number of 7×10^4 which is in the range of the Abramovich figures wherein he discounted the effect of N_{Re} . Accordingly, the value of K of 1.41 used with Equation (4) is consistent with the theoretical work and experimental references used by Abramovich for air jets.

It is, therefore, recommended that the following relation be used for axial velocities for a circular jet:

$$u_r = (u_x/u_0) = 1.41 N_{Re}^{0.135} (D_0/x) \quad (5)$$

NOTATION

a	= constant
D	= diameter, m
K	= constant
m	= meters
N_{Re}	= Reynolds number Du/ν
s	= seconds
u	= velocity, m/s
x	= distance along axis from jet origin
ν	= kinematic viscosity, m^2/s
θ	= angle of expansion of jet

Subscripts

m	= maximum
0	= at jet origin
x	= distance along axis from jet origin
30	= 30 jet diameters from origin

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High Gradient Magnetic Separation In A Viscous Flow Field

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The basic equations governing the motion of a small paramagnetic particle in a high gradient magnetic separator (HGMS) were derived in an earlier paper (Cummings

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et al., 1976). The purpose of this note is to extend the previous theory by modeling the fluid flow with Lamb's solution and by including the close range London forces in the force balance.

A HGMS usually consists of a solenoidal electromagnet which has its interior packed with thin metal strands or